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ARCHITECTURAL HISTORY AND THEORY

Reconsidering a proportional system of timber-frame structures through ancient mathematics books: a case study on the Muryangsujŏn Hall at Pusŏksa Buddhist Monastery

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ABSTRACT

The mathematics references in ancient China, the *Zhoubi Suanjing* and the *Jiuzhang Suanshu*, present information on formative ideas of ancient people and their perception of objects. The introduction to the *Yingzao Fashi* mentions mathematical sources, including the *Zhoubi Suanjing*. Both of these books focus on the philosophical concept of *Tianyuan difang* (Heaven is round and Earth is square), as well as inscribed and circumscribed circles. The square root of 2($\sqrt{2}$), which can be derived from this part, proves to be an essential criterion for building, seen in Korea, China, and Japan. Using the exemplary Koryŏ building, the Muryangsujŏn Hall at Pusŏksa Buddhist Monastery, this thesis shows that the standard ground plan width of the outermost bay has a $\sqrt{2}$ ratio to the central bay width. Its cross-section, likewise, proves that $\sqrt{2}$ times or twice the distance or height (relying on the height of the eave columns) are applied to the distance or height between each column and purlin in the application of arithmetic and geometric concepts. In the future, this work will be a reference for the reconstruction design of ancient buildings prior to the Koryŏ period, analogous to the Muryangsujŏn Hall.

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Muryangsujŏn Hall at Pusŏksa Buddhist Monastery; $\sqrt{2}$; *Zhoubi Suanjing*; *Jiuzhang Suanshu*; *Yingzao Fashi*

1. Introduction

1.1. Research background

Studies of timber-frame systems used in East Asian wooden buildings have been conducted for many years, occupying a very important position in the architectural history of Korea, China, and Japan. Chinese architecture in the Tang, Liao, Jin, and Yuan Dynasties, in common with that of the Ming and Qing eras, produced architectural dignities. In Korea, there are three building styles based on a bracket system (wooden structural elements fitted to the tops of columns or beams, in order to support the weight of roof eaves), which were prevalent in the Koryŏ period (918–1392): 1) the *chusimp'o* 柱心包 style, which placed the bracket complex directly on the column head, 2) the *tap'o* 多包 style, which included an inter-columnar bracket complex besides those on the column heads, as well as spaces between the brackets (the dominant architectural style of the Chosŏn period), and 3) the *ik-kong* 翼工 style, a bracket complex that featured simplified, beak-like protrusions on highly important buildings. Such bracket systems have been developed into various shapes over time.[1]

The ground plan determines structural forms and techniques used to build an interior timber-frame structure, and the uses and positions of the inner elements, such as the purlins, beams, and columns, are of great significance for structural stability. In addition, the scale is an important factor in deciding the size or location of a wooden building. The decision regarding the position of purlins and columns is stemmed from the ground plan. Depending on historical periods, there have been diverse positions and sizes of beams and girders, as well as various methods of integrating bracket complexes to interior framed structures. It appears that there were certain rules for construction; however, the earliest ancient documents that apply to the design concepts of wooden architecture on the Korean Peninsula are records of construction following the mid-Chosŏn period. There is little information or remaining pieces of buildings dating from earlier times. This is a difficult aspect of the search for formative ideologies or restoration designs in the Paekche, Silla and Unified Silla periods during the pre-Koryŏ era. This study, therefore, attempts to derive these concepts from East Asian mathematics texts, which provide a glimpse of ancient formative thinking, as well as the associated figures, and the proportional system united with these figures.

1.2. Existing research problems

In order to restore ancient Korean architecture and urban landscapes of the Silla, Unified Silla, and Paekche periods, reconstruction designs rest on the building methods

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[1]The McCune-Reischauer system of Romanization generally is used throughout this thesis, with some exceptions, especially for earlier common usages of Korean names, such as Cha, Kim, Han, etc.

described in the *Yingzao Fashi* 營造法式 (Treatise on Architectural Methods, 1103 CE)[2] and then compared to other extant Koryŏ buildings. There are few mathematical models of the proportional system associated with wooden architecture, and much doubt has been cast on the reliability of reconstruction designs. However, some Chinese researchers lay the groundwork for reconstruction and restoration designs in the future. First of all, Chen Mingda, referring to the proportions of √2:1 or 3:2, obtained the ratio of the horizontal and vertical sections of the first, balcony substructure, and second floors while analysing the example of Dulesi Monastery's the Guanyinge Pavilion (Chen 2007, 16). Wang Guixiang has proposed a valid argument for a 1: √2 ratio of the height of the eaves columns (exterior columns or perimeter columns, named *yanzhu* 檐柱 and *weizhu* 外柱 in Chinese) and the height of the eaves purlins (Wang 2011a, 2011b).[3] (Figure 1) The √2 description, along with the formative thinking of ancient people, is not enough; an explanation of the proportional system of an interior timber-frame structure remains very vague. Details are lacking in the connection between the ground plan and scale. Dongdadian (the Great East Hall) at Foguangsi Monastery, Wutaishan, Shanxi, is among the oldest buildings in China and does not follow the 1: √2 rule dictating the ratio between the height of the eaves columns and the height of the eaves purlins. Recently, Wang Nan's studies shed new light on the subject. He sketches out the proportional relationship between square and circle drawings with the ratio of √2 times in that the *Zhoubi Suanjing* and *Jiuzhang Suanshu* include the contents of the round-square and square-circle maps, and the principle of "tianyuan difang." Wang's researches confirm that the square-circle diagrams with geometric rapport are commonly applied in designing ancient capital cities (from the Xia to the Qing Dynasties), the layouts of important landmark building complexes for the central axis of Beijing, forty-one Buddhist pagodas, the Great East Hall at Monastery Foguangsi, the Guanyinge Hall and the Shanmen Gate at Dulesi Buddhist Temple in Liao architecture, and palace buildings (the Forbidden City during Ming-Qing Dynasties).[4] They all embrace the ratio of the

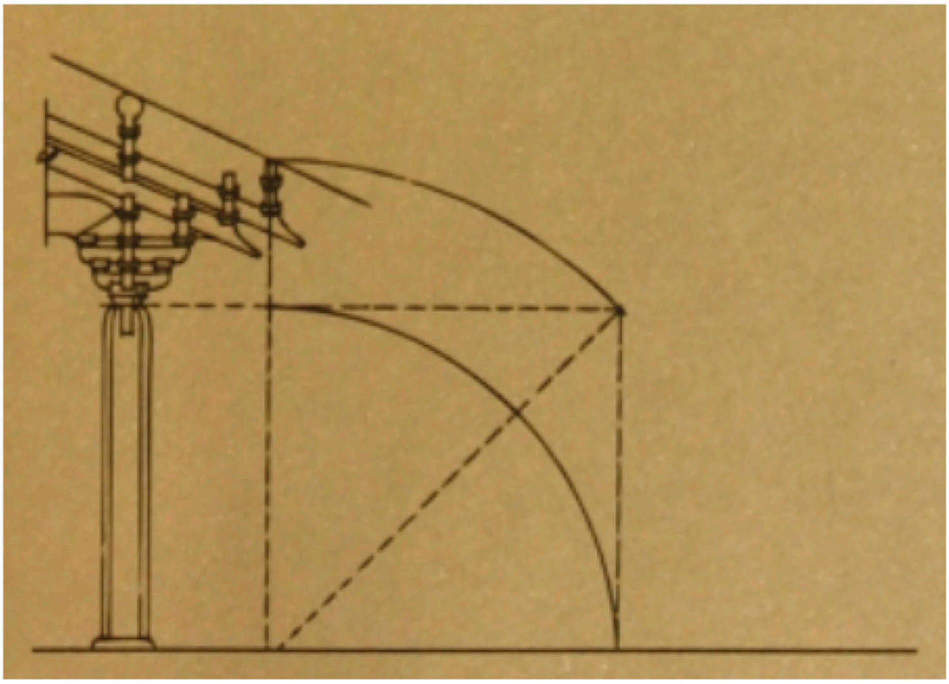

Figure 1. A line drawing of proportional system from a Wang Guixiang book cover.

[2]Although we do not sure how the *Yingzao Fashi* has influenced Korean architecture, a few researchers attempt the reconstruction plan assuming the impact on the *Yingzao Fashi* in Korea. Among them, Kang and Yun proposes a reconstruction plan about the Monastery Pulguksa's Kŭngnakchŏn Hall, drawing on document data from the *Yingzao Fashi* and archaeological remains about wooden structures left in Korea, China and Japan (Kang and Yun 2006, 217–218). Another application of the *Yingzao Fashi* has come to focus on the restoration plan of the corridor territories at the Mirŭksa Buddhist Temple, although the monastery's foundation does not accord with the publication of the *Yingzao Fashi*. The research accompanied by fellow researchers at the National Research Institute of Cultural Heritage in Korea is animated by two appropriate assumptions. They argue that the restoration plan can produce a compromise through in-depth contemplation to the *langwu* 廊屋 (roofed corridors linking main buildings) in the *Yingzao Fashi*'s "Damuzuo zhidu" (the major carpentry system) which contains the proportional system coupled with building grades, and, admittedly, they opine that it can come up with a settlement through the utilization of proportional systems employed for the restoration plans of roofed corridors at other extant Buddhist temples such as Horyuji 法隆寺, Yamadadera 山田寺, and Mikawa Kokubunji 三河國分尼寺 (Kungnip munhwajae yŏn'guso 2018, 80–88).

[3]Wang's two articles give a good account for inscribed and circumscribed circles as examples of the *Tianyuan difang* in ancient literature, as well as the conjunction between 1 and √2 ratio, which are found in the Kanxiang, the beginning section of the *Yingzao Fashi*.

[4]For reference, Fu Xinian, a Chinese scholar, underlines that the concept of geometric centrality has a long history in China through interesting analysis of the modularity and systematic planning of palace and state temple architecture, site layouts. In the choice of the square grid system for a long history which depends on the importance and size of the architectural cluster, Fu argues that Daming Palace and Luoyang Palace in the Tang capitals Chang'an and Luoyang, respectively, used a 50-zhang-square grid that was further developed at Yuan Dadu and Ming-Qing Beijing, punctuating the use of different scales and modular proportions for different building sizes and complexes. (Fu, Steinhardt, and Harrer 2017, 329–361; Fu 2004, 319–355).

$\sqrt{2}$ and $\frac{\sqrt{3}}{2}$ times in proportional treatment, which incorporate the intimate associations with the circle-square and square-circle diagrams. His results show that the geometric ratio can be utilized in planning and designing cities and buildings through actually measured and restored drawings. (Wang 2017a, 2017b, 2017c, 2018a; Wang 2018b, 2019)

Another research has explained the 1: √2 relationship for ancient building constructions in the Asuka Period. On the basis of the ratio 1: √2, the study proves the employment of design methods, which define the distance from the pagoda foundation to the diagonal line, and its correlation to the depth of the halls, showing that Japanese craftsmen made use of a right angle ruler. (Ono 1964, 623 –, 625) Likewise, major tools for drawing and measurement are depicted in surviving examples of illustrated religious materials such as the images of Fuxi and Nüwa in the Wuliangci bas-relief or Astana Tomb, which show them holding an L-square ruler and a compass (Figure 2). Although the philosophical background and evidence of the √2 application are still insufficient in building construction terms, they offer a key mechanism to an understanding of ancient design rules in East Asia resulting from the √2 implementation as a key principle.

1.3. *Research methods and scope*

This paper provides an in-depth discussion of the square root of 2[5], using mathematics and astronomical books, such as the *Jiuzhang Suanshu* (The Nine Chapters on the Mathematical Art) and the *Zhoubi Suanjing* (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven), which was written by an anonymous author around the 1st century BC. Korea's ancient history book, the *Samguk sagi* (History of the Three Kingdoms), mentions in the chapter seven of the Silla government offices in Book 38 that the *Jiuzhang Suanshu* was used in the 7th-8th centuries as a regular mathematics textbook in the Sanhak department 算學科 (arithmetic education) at the Silla-era Kukhak 國學 (the state-run educational institution). It is possible that the *Jiuzhang Suanshu* entered the peninsula at this time. The Nine Chapters of the *Jiuzhang Suanshu* examine *Gougu* 勾股 (base and altitude), in close association with the *Zhoubi Suanjing*. Liu Hui comments on the first three problems of the chapter, which use the case of base 3, altitude 4 and hypotenuse 5 to illustrate how each side may be found using the other two. Specifically, Liu explores instructions for finding the square root of the hypotenuse from the sum of the squares of the other two sides (Cullen 2007, 88).

In this overall perspective, this thesis intends to ferret out the proportional systems of the interior timber-frame structures at Pusŏksa Buddhist Monastery 浮石寺, which is considered to be one of the oldest buildings in Korea. So far, no surviving wooden buildings constructed during the Three Kingdoms period have been discovered in the Korean Peninsula. Among the buildings left, the Kŭngnakchŏn Hall 極樂殿 of the Pongjŏngsa Buddhist Monastery 鳳停寺 is believed to be the oldest in Korea.

1.4. *Pusŏksa Monastery and its Muryangsujŏn Hall*

The Muryangsujŏn Hall 無量壽殿 of Pusŏksa Monastery 浮石寺 was initiated by Ŭisang 義湘 (625–702), the purported founder of Hwaŏm in Korea. The Pusŏksa Monastery was not as large as it was at the time of its foundation, estimating that the monastic complex developed into a large-scale temple in the 9th century, the late Unified Silla. In early Koryŏ times, it was said that Wŏnyung guksa 圓融國師 (964–1053) preceptor reformed the Muryangsujŏn Hall in the 7th year of King Hyŏnjong (1016) (Munhwajaegwalliguk 1976). According to the *Pongwangsan pusŏksa gaeyŏn'gi* 鳳凰山浮石寺改椽記 (Repair Records at Monastery Pusŏksa in the Phoenix Mountain), since then, the Muryangsujŏn Hall was burned by Japanese pirates during the 7th year of King Kongmin's reign (1358) at the end of Koryŏ Dynasty. In the 21st year of King Kongmin (1372), Wŏnŭng guksa 圓應國師 (1307–1382) preceptor was appointed as the Pusŏksa's chief priest at the king's command, and in the second year of King U (1376) during Hongwu 9, he repaired the Muryangsujŏn Hall. These evidences were confirmed by explanatory legends uncovered in 1916 during the dismantlement and repair works of the Muryangsujŏn Hall.[6] (Cultural Heritage Administration of Korea 2002, 63–65)

[5]The square root of 2 is an irrational number. It cannot strictly be measured with a ruler based on our unit of measurement, no matter how small we mark fractional subdivisions. However, when we calculate the length of the diagonal of a rectangle, employing the Pythagorean Theorem, we obtain, indirectly, an irrational number (Carnap 1995).

[6]Two records on the Muryangsujŏn Hall's repair works in 1016 and 1376 were found within the corner bracket set at the northwest, and on wooden members at the southwest corner toward the front facade.

The former inscription is written as follows:

此寺唐高宗二十八年儀鳳元年新羅王 命義相法師始立創建 後元順帝十七年 至正戊戌敵兵火其堂 尊容頭面 飛出烟焰中 在于金堂西隅文藏石上 而奏于上 洎洪武九年丙辰 圓融國師改造改金 而至于萬曆三十九年 辛亥五月晦日風雨大作柝其中樑 明年□壬子 改椽新其畵彩儼若旧制也 記其匠碩及勸緣人以示後也.

The latter is written as follows:

此　堂自洪武九年經倭火後改造而至萬曆三十九年自折衝椽也壬子年始役畢於癸丑年八月也.

They record that the building was reformed by Wŏnyung 圓融 guksa preceptor in 1376. However, it is believed that the chief monk of the Pusŏksa was originally Wŏnŭng 圓應 guksa preceptor at that time. To put it exactly, the name Wŏnyung was a wrong record, and all inscriptions on repair works drawn up in 1612 ~ 1613 (in the 40th to 41st year of the Wanli era, i.e. the 4th to 5th year of King Kwanghae) present a time lag of about 240 years from 1372. (Munhwagongbobu Munhwajaegwalliguk 1980, 16).

Figure 2. Painting of Fuxi and Nüwa holding a compass and an L-square ruler, 7th century Astana Tomb in Turpan, Xinjiang Uygur, China.
Source: National Museum of Korea.

Great attention has been shown to the question regarding the range of the 1376 repair work. Not only is the inscriptions recorded about 240 years later, but both the extent of the fire loss and the magnitude of the reform are obscure in that the two records emphasize 'recomposing a building and repainting a statue with gold simultaneously 改造改金' and "remodeling a building 改造," rather than its reconstruction.

Such being the case, there have been many observations that the architectural forms and techniques on the Muryangsujŏn Hall seem to utilize much older methods than those shown in the 14th century's buildings (Han'gukpulgyoyŏn'guwŏn 1988, 88). In comparison with the architectural features of the Chosadang Hall situated in the same monastic site, the academic generalized perspectives guess that the Muryangsujŏn Hall was constructed 100 to 150 years earlier (Sekino 1941, 733; Munhwagongbobu Munhwajaegwalliguk 1980, 150; Chŏng 1974, 27–32).

Likewise, there are interesting researches in comparison to other extant examples in the Koryŏ age such as the Monastery Pongjŏngsa's Kŭngnakchŏn Hall, the Monastery Sudŏksa's Taeungjŏn Hall, and ancient Chinese buildings, Han argues that the modification in 1376 was about repairing some of the lost parts at the time of the fire, and there are claims that the existing Muryangsujŏn Hall retains its original status as it was when the preceptor Wŏnyungguksa reconstructed the building in 1016 (Han 2002, 148). Cha's research points out that the Muryangsujŏn Hall's diagonal beams joined with corner bracket clusters are similar to the construction method of pre-11th century wooden buildings in northern China, as its universal applications are mainly seen through the examination of edifices built in the Tang (Foguangsi Monastery's Todaidian Hall, built 857), the Song (Yongshousi Temple's Yuhuagong Hall (destroyed), built 1008; the Huayansi Monastery's Bhagavat Sutra Repository built 1038), and the Liao Dynasties (Monastery Dulesi's Guanyinge Pavilion, built 984), and gradually degenerated after the Jin Dynasty (Cha 2014, 131–142). Kim Dogyŏng notices the problem that wooden members which consist of the column-top bracket clusters and column-top tie beams to make a timber-framed structure keeps the same size with regular measurement, asserting the remarkable assumption that the Muryangsujŏn Hall might be partly repaired in the 14th century, cherishing its original shape in the 11th century (Kim 2014, 152). Kim Youngjae recognizes that the Muryangsujŏn Hall upholds old construction methods in two regards: the first, the Muryangsujŏn Hall does not serve the extension of architrave through column top at the corner, while the extant buildings constructed during the Song and the Liao Dynasties across China have architraves that pass through the pillar tops along both building axes; the second, on closer inspection, the Muryangsujŏn Hall's stone foundation platform with subsumed footings for round columns follows the construction method in the Unified Silla period since it bears a resemblance to the developmental stages that imitate the outline of timber frameworks in performing masonry work (Kim 2011, 306, 447).

During the Japanese forced occupation of Korea, moreover, Sekino Tadashi identified the Pusŏksa Monastery's Muryangsujŏn Hall as the oldest building on the Korean Peninsula. In 1911, Sekino wrote that he first discovered wooden architecture built in the Koryŏ era in an article. He made a brief report about the location and history of Pusŏksa Temple, along with its construction age and architectural value of the Muryangsujŏn and Chosadang Halls (Sekino 1913). He praised the two buildings, commenting "the Muryangsujŏn and Chosadang Halls are indeed very artistic and skilful buildings in the Koryŏ period, and the traditional multi-coloured paintwork of the buildings was truly ecstatic. I (Sekino) believe that they are enough not only to be the oldest known buildings discovered but also to regard it as the greatest exquisite beauty among wooden buildings in the Koryŏ times." Sekino evaluated the architectural value of the Pusŏksa Muryangsujŏn Hall, explicating "it is an excellent building analogous to the masterpieces of the Kamakura period. The construction style is particularly commensurate with ancient Japanese architecture. The internal framework built above bracket-sets and the underside of the tiled roof bears a striking likeness to the Tenjiku (Daibutsu) style in the Kamakura era, and the shuttle-shaped columns with entasis are similar to those in the buildings at Horyuji Monastery." (Sekino 1913, 3-5) Likewise, according to the field cards recorded by Sekino in 1913, one year after the investigation of Pusŏksa Temple, he found the Ŭngjinjŏn Hall at Sŏgwangsa Monastery, another wooden building in the Koryŏ era, in Kangwŏn-do Province (Sekino 1941, 645–650). Sekino recognized the above-mentioned three buildings would be constructed in the Koryŏ era (Sekino 1941, 645–650, 723–742). After that, apropos the construction era and building characteristics of the Muryangsujŏn and Chosadang Halls, On 27 October 1923, in the Chōsenshi gakkai 朝鮮史學會 (Chosŏn History Society), he announced a paper on the subject of "Chōsen saiko no mokuzō kenchiku [Korea's oldest wooden building]" His thesis noticed the difference between the architectural styles of the two buildings and concluded that the Pusŏksa Muryangsujŏn Hall was at least 100 to 150 years older than the Chosadang Hall 祖師堂 (a hall of the founder). Sekino confirmed, in a manuscript excavated during the repair work of the Chosadang Hall, that the Chosadang Hall was built in the third year (1377) of King U in the Koryŏ Dynasty and was repaired in the 21st year (1490) of King Sŏngjong in the Chosŏn Dynasty, although he mentioned the needs of additional comparative studies with other extant Korean buildings in the late Koryŏ and the early Chosŏn Dynasties. Sekino Tadashi asserted that the Muryangsujŏn Hall, which had no comparable examples in Korea and had no contemporary account of that time, was built in the early 13th century.[7] (Sekino 1923, 1941, 723–742)

In the light of these considerations, the Muryangsujŏn Hall which will be discussed in this thesis, is strongly believed to have been built between the 11th and 13th centuries in the fact that the Muryangsujŏn Hall employs significant structural elements such as architraves without any extension through the columns at the corners, diagonal beams merged with corner bracket sets, the repetitive utilization of regular size members, as well as the acceptance of the pre-Koryŏ's methods to establish a stone foundation platform.

Hence, this research examines an arrangement of foundation stones on a ground plan, together with an interior timber-frame structure, with a focus on the Muryangsujŏn Hall at the Pusŏksa Buddhist Temple. This thesis delves into fundamental design principles with proportional systems in that all columns inside the building are well arranged at regular distance without their reductions and movements, keeping an old construction method, and in that proportional regulation in the application of the square root of 2 (√2) for its construction are proportionate to those of Chinese buildings in the contemporary period. The following studies on the Koryŏ buildings should be more produced as future research tasks.

2. The Yingzao Fashi, and the mathematics books in ancient China, the Jiuzhang Suanshu and the Zhoubi Suanjing

Importantly, the imperial Northern Song dynasty (-960–1127 CE) treatise on architecture, the *Yingzao Fashi* 營造法式 (Treatise on Architectural Methods, 1103 CE), contains a series of numerical values and terminology for constructing buildings. At its head, the *Yingzao Fashi* has a table of contents and the foreword, "Kanxiang 看詳" (Examination of details). The treatise classifies all architectural work into thirteen systems recorded in thirteen chapters, including the Damuzuo 大木作 (structural carpentry and woodwork) and Xiaomuzuo 小木作 (joinery and non-structural carpentry). In the "Kanxiang," what is unusually noteworthy is that various ancient documents associated with mathematical ideas are presented. Among them, concerning the application of the square root of 2 (√2), the Qujingwei 取徑圍 (geometrical tie-up between diameter and circumference) proposes the fundamental rule about the approximate rate of the slanting length, "If one side of a square is 100 in length, its diagonal length is 141 as the numerical value. 方一百，其斜一百四十有一," following the numerical principles in the *Jiuzhang Suanshu*, criticizing that the "square seven oblique ten 方五斜七 (if one side of a square is seven in length, the diagonal length is ten as the numerical value)" reflects lots of negligence (Li 1103 (Song), 22), although the design formula "square seven oblique ten" in the combination of rational numbers is more precise and closer to the integer ratio of √2, an irrational number. Such design tradition which had been long preserved by professional builders solves for easier building constructions the problem of √2 times unable to be strictly measured with a ruler grounded on the unit of measurement.

Likewise, certain contents of the *Zhoubi Suanjing* are included as follows: "Shang Gao answered. Numbers and their law – arithmetic – derive from the circle and the square. The circle arises from the square, the square from the carpenter's L-shaped try square, and the carpenter's try square from multiplying nine by nine and getting eighty-one. The myriad things, each meeting their roles, are measured by the circle and the square; whereas the chief architect, to create models and styles, has devised the pair of compasses and the carpenter's try square. He impairs a square to make a circle or breaks a circle to make a square. A circle that fits in a square is called an inscribed circle in a square, while a square that fits in a circle is called a circumscribed circle.[8]

This shows that the *Yingzao Fashi* considers the *Gougu yuanfang tu* 勾股圓方圖(Drawing for bases, altitudes, circles, and squares) in the *Zhoubi Suanjing*. (Figure 3) Furthermore, the dialogue of Shang Gao in the *Zhoubi Suanjing* includes an in-depth discussion on the relationship between circles and squares, along with the concept of *Tianyuan difang* (Heaven is round and Earth is square). The text reads, "The square belongs to the earth, and the circle belongs to the sky; the sky is round and the earth is square. The number of the square is basic, and the circle comes from the square (方屬地, 圓屬天, 天圓地方. 方數爲典，以方出圓)." In accordance with Zhao Shuang's commentary, this means that the earth is stationary while the sky revolves in motion, a cosmological concept universally shared by the ancients (Chen 1984, 95). The sentence ends with, "So whoever knows the earth is wise, and whoever knows the sky is holy. Wisdom comes from

[7]An article with the same subject as the lecture was published in the same year. Sekino Tadashi, 1923. "Chōsen saiko no mokuzō kenchiku [Korean oldest wooden building]," *Chōsen to kenchiku* [Chosŏn and Architecture], Vol 2, No. 8, Chosŏn Architecture Association. The paper was also subsumed in a book, which was compiled in 1941. Sekino Tadashi, 1941, *Chōsen no kenchiku to geijutsu* [Korean Architecture and Art], Tokyo: Iwanami Shoten.

[8]昔者周公問於商高曰: 數安從出？商高曰: 數之法出於圓方. 圓出於方. 方出於矩, 矩出於九九八十一. 萬物週事而圓方用焉, 大匠造製而規矩設焉. 或毀方而為圓, 或破圓而為方. 方中為圓者謂之圓方；園中為方者謂之方圓也. (Li 1103 (Song), 19–21).

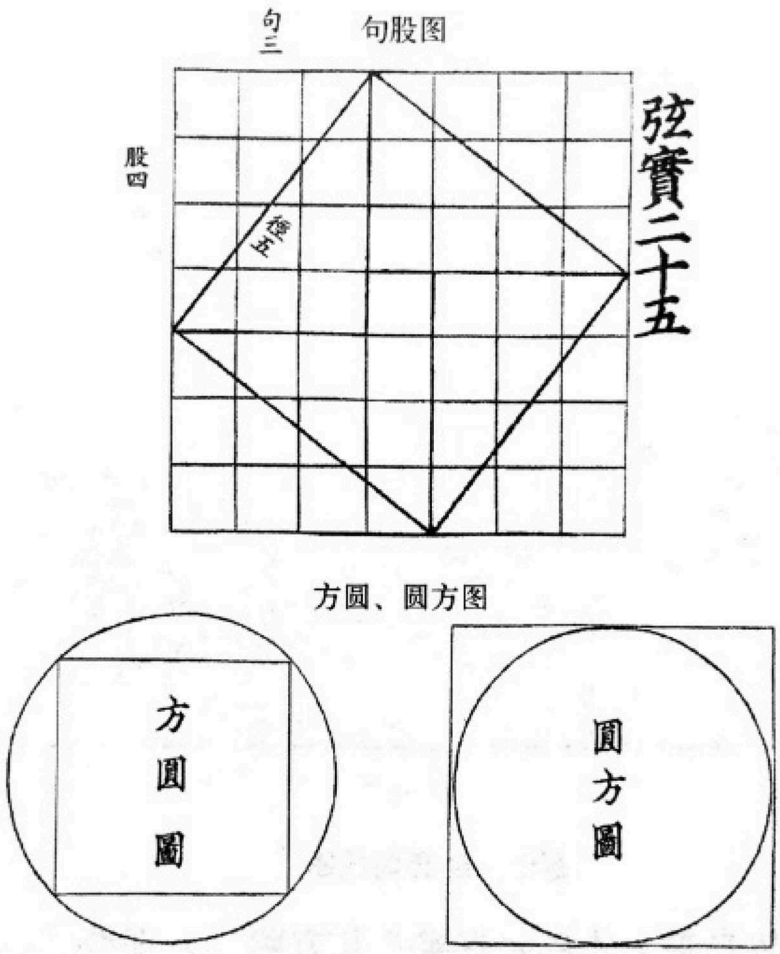

Figure 3. Line drawings of Gougutu by Shang Gao (revision version) and Fangyuan Yuanfangtu [Rounded-Square, Squared-circle Map] by Shang Gao (restoration version) (Cheng and Wen 2012).

the base, and the base comes from the bend. It controls everything through digitizing it by using a curved line (是故, 知地者智, 知天者聖。智出於句, 句出於矩. 夫矩之於數, 其裁制萬物, 惟所爲耳)." The above-mentioned concept (Figure 3), which considers squares and circles on a mathematical basis, is similar to the description of the *Gougu yuanfang tu* (Drawing for bases, altitudes, circles, and squares) in the *Yingzao Fashi* of the Song dynasty. It illustrates how combining squares and circles can control myriad things, which can be converted into various things (Cheng and Wen 2012, 12). Likewise, the *Jiuzhang Suanshu*, another ancient mathematics book, explicates the L-shaped try square and compass as measurement devices that can embody innumerable things (Shen et al. 1999, 520-522). In line with the preface commented by Liu Hui in the 3rd century, the book evolved from the *Jiu shu* 九數 (Nine Operations with Numbers), which dates from the time of the Duke of Zhou in the 11th century BC. It describes a mathematical method similar to that of the *Zhoubi Suanjing*: "Even those results delivered by the law handed down through the generations should be presented by measuring the length with a compass and an L-shaped try square (至於以法相傳, 亦猶規矩度量, 可得而共)." The law (*fa*) sets out mathematical principles, the regulation (*gui*) refers to a compass drawing a circle, and the rectangle (*ju*) refers to an L-shaped try square making a square – these represent the tools used to draw circles and squares individually. The weights and measures (*duliang*), on the other hand, represent a standard unit of metrical type that calculates the length, width, depth, volume, and capacity of myriad things in the world, and reflects numerical rapports between innumerable things. In other words, standards and measurements always represent the formats of space and quantitative ties. In ancient times, in China, geometry was associated with number and amount, and all matters were solved through arithmetic and algebra (Guo 2009, 7). Its introduction also indicates that a change in worldview expounds the circumstances of myriad things. "Fu Xi painted the eight trigrams (*bagua*) in remote antiquity to communicate the virtues of the gods and parallel the trend of events in the earthly matter, and then he invented the nine-nines algorithm (*jiujiu*) to co-ordinate the variations in the hexagrams (*liuyao*) (昔在庖犧氏始畫八卦, 以通神明之德, 以類萬物之情, 作九九之術, 以合六爻之變)." Led by the virtues of the gods, mathematics became a symbolic language, its main function being to classify the state of all things (Guo 2009, 2). In this way, the key idea in both books is that myriad things in neighbouring areas can be created using circles and squares. The diagram of an inscribed circle and a circumscribed circle in Figure 2 shows that circles and squares are the basic components for construction.

Furthermore, Liu Hui states that the Yellow Emperor transforms and extends the cosmic principle of the trigrams tremendously to solve practical problems, such as divination, regulation of the calendar, and harmonization of the musical scale. The prefatory chapter "Kanxiang" of the *Yingzao Fashi*, as has been noted earlier, mentions various phrases that relate to the length of figures and buildings. The parts of rectangles are as follows: "If the lateral side of a square is one hundred in length, its diagonal line across the square is one hundred and forty-one in length (方一百其斜一百四十有一)." In this passage, the square root of 2 (√2), or the length of a diagonal line across a square, is notable. The simplest mathematical form of √2 is an equilateral triangle. Assuming that the equilateral triangle's base and height measure 1 at right angles, the length of the hypotenuse of the triangle is defined as the square root of 2 or 1.414 when expressed as a numerical value in modern mathematics. A square results from combining two equilateral triangles. The method prescribed in the *Yingzao Fashi* to determine the length of figures is based on the selection of the Pythagorean Theorem.[9] The formative ideas of the ancients, in consort with the composition principle of circles and squares, which are embedded in the *Zhoubi Suanjing* and the *Jiuzhang Suanshu*, permeate the contents of the *Yingzao Fashi*.

3. The proportional system of timber-frame structures in the Muryangsujŏn Hall at Pusŏksa Monastery

Pusŏksa, located in Yŏngju City, North Kyŏngsang Province, is a Buddhist monastery established by the monk, Ŭisang, and one of ten monasteries grounded on Hwaeom (*Avatamsaka*) thought. The Muryangsujŏn (the Hall of Immeasurable Life) was constructed as the main hall with a hip-and-gable roof in the *chusimp'o* style, following bracket complexes placed only at the heads of the building's structural columns. As aforementioned, the building was completely dismantled and then reassembled for the repair work in 1916 during the Japanese forced occupation of Korea .[10] Korean researchers believe that the interior timber-frame structures were much transformed by the repair work (Han 2002, 139–48). In the main hall, the structural system of a diagonal beam in the corner of buildings, which included a bracket complex and column, is tantamount to the side frame structure techniques of the Tang and the Liao periods. The hall has attracted attention due to research that identified construction techniques from before the 12th century, which correspond to northern East Asian architecture in the contemporary period (Cha 2014, 131–142, 2016, 78–103). But, the Muyangsujŏn Hall experienced a typical design process in the fusion with the southern and the northern architecture of China[11] (Kim 2011, 538–540). It is a five-by-three–bay building with a front façade and side façade measuring 18,751 meters and 11,511 meters, respectively. There are no inner columns that have been migrated or removed. In particular, both the building platform and foundation stones that are presently visible (the natural interior foundation stones are elements that were replaced during the Japanese forced occupation) are a variation that is rarely seen after the Unified Silla period. Some of the original styles are inherited by recycling elements previously used in the Koryŏ period at the time of its construction. (Figures 4 and 5)

On the ground plan of the Muryangsujŏn Hall at Pusŏksa Monastery, the central front facade bay and the bays on either side of the central bay along the front facade are the same widths, while the outermost bay is narrower than the other three.[12] (Figure 6) There are some exemplary buildings in East Asian architecture in which the ratio of the central bay to the side bays is 1:1. To put it briefly, the 7th century main hall at Daikandaiji Monastery (Nara), the late 8th century main hall at Kamŭnsa Monastery (Kyŏngju, North Kyŏngsang), the 10th century main halls of Buddhist Monasteries in Parhae, the early 11th century main hall at Fenguosi Monastery (Yixian, Liaoning), and so on (Kim 2011, 304–312). The side facade of the building is a timber framework with two smaller bays on either side of its central bay. The inside of the building is therefore relatively wide. The interior stone plinths and columns are arranged on the same line as the external elements. Previous studies[13] have generally had a strong tendency to regard a central bay as an absolute standard for framing a wooden building, but this paper offers

[9]Liu Hui describes the ratio of diagonal length in the octagonal geometrical construction method given in the *Jiuzhang Suanshu*, and argues that the error of the square root of 2 (√2) is approximately 1% (Liu 2014).

[10]The repair works of the Chosadang and the Muryangsujŏn Halls were performed during four years from 1916 to 1919 at a cost of 23,566 *yen* (Kim 2011, 440).

[11]Kim maintains that the Pusŏksa's Muyangsujŏn Hall is a typical building combined with directly opposed notions between the south and the north of Chinese architecture; in the south china the one is the acceptation of *dingtougong* (half-bracket arm) with a tenon-and-mortise to stabilize the combination of the longitudinal with the transverse beams and the columns of the building; in the north China the other is the employment of *tuojiao* (inclined struts) between one transverse beam and another transverse beam in comparison with the construction method of the Pongjŏngsa's Kŭngnakchŏn Hall (Kim 2011, 539).

[12]The width of the central bay and the side bays are slightly different, but they are all about 4,200 mm long (Cultural Heritage Administration of Korea 2002, 12).

[13]Interesting suggestions for the usage of the central bay are offered by some researchers as both a functional space and a significant factor to make room for monks and laypersons. Kim and Pak note that, since the 7th century (the Unified Silla), as the function of the central bay becomes more important over time, the central bay tends to be wider than the outermost bay on the ground plan (Kim and Pak 2008, 20–21). Kim Youngjae puts forward the very interesting hypothesis that the particular employment through the long width of bays might be intended to express the authority as a Buddha hall, and it might help adherents in a sense of awe contemplate the Buddha images at a long distance (Kim 2011, 309). Additionally, through the comparative

Figure 4. Front view of the Muryangsujŏn Hall at the Pusŏksa Monastery.

Figure 5. Interior timber frame structures of the Muryangsujŏn Hall.

a new interpretation in the form of an arithmetical commentary, which is contrary to existing views. In the Muryangsujŏn Hall at Pusŏksa Monastery, depending on the direction in which columns and eaves purlins are placed, and assuming that the width of the outermost bays is one (1) and that the location of the corner columns is regarded as a central point to draw figures, the authors can draw squares or circles, and the lateral length of a square can be considered as two (2). As seen in Figure 6, based on the center point of the corner columns, both an inscribed circle with a radius of one (1) and a circumscribed circle with a radius of the square root of 2 (√2) can be drawn on the ground plan. The inscribed circle meets the second column to the left and the corner edges of the building platform when looking at the southern front facade, while the circumscribed circle matches the position of the interior corner columns. All columns in the design of the building are arranged at intervals of √2 times, assuming the width of the outermost bay is one (1). In contrast to the

analysis between extant middle-gates at ancient temple sites and historical remains, Han argues that the restoration plan for the middle-gate's archaeological remain at Mirŭksa Monastery can be carried out looking upon the central bay on the ground plan as an important factor (Han 2012, 206).

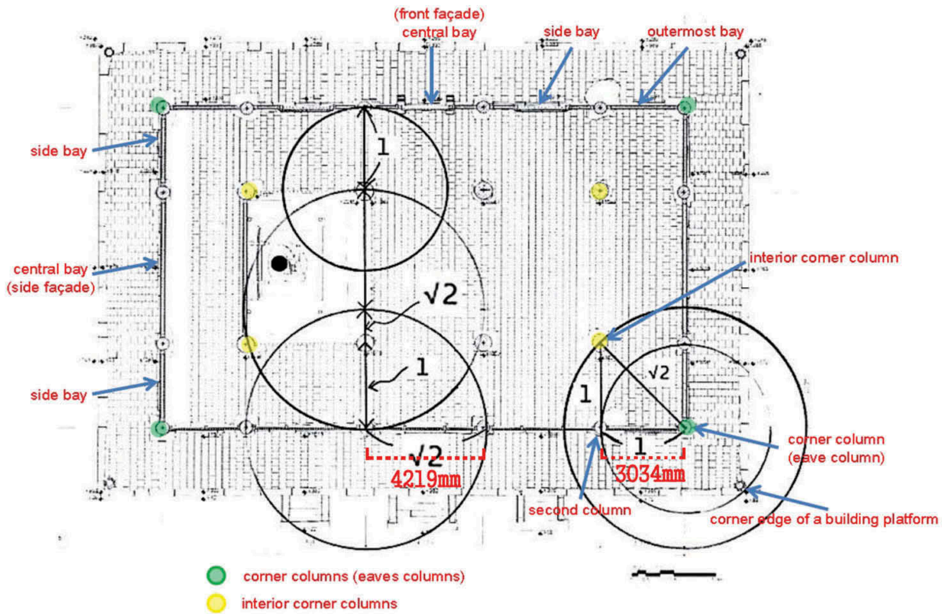

Figure 6. The ground plan of the Muryangsujŏn Hall (Cultural Heritage Administration of Korea 2002, 121).

common building standards of the timber framework, this building demonstrates that the width of the outermost bay is the basis of the column arrangement, not that of the central bay. On the contrary, despite a few errors in the placement of roof purlins, on a closer look at the side façade of the building in the direction of the transverse beams that form the roof, the distance between the side and central bays at the side façade is tuned to a ratio of 1: √2 + 0.414, on the assumption that the distance between the eaves and the interior columns is one (1). This numerical value is understood as an arithmetical concept rather than a geometric concept. Furthermore, the width ratio of the outermost bay to the central bay or the side bay toward the front façade is 1: √2. This shows that the proportional system in the synthesis of 1 and √2 is applied to the interior ground plan. The outermost front façade bays are 3,034 mm long, and the central bay is 4,219 mm long. √2 times the length of the outermost bays is 4,290 mm. This shows a margin of error of about 1.7 percent from the actual measured length of 4,219 mm (Cultural Heritage Administration of Korea 2002, 128). In addition to the arrangement of the columns on the ground plan, by looking carefully at the tie-up between the building platform and columns, it can be seen that the distance from the corner edge of the building platform to the corner columns is almost the same as the width of the outermost bay. The ratio of the distance between the corner edges of the building platform and the corner columns to the distance between the corner eaves columns and the interior corner columns is 1: √2.

As seen in Figure 6, it is highly probable that the ratio of 1 to √2 is applied to define a ground plan that situates columns, and the width and depth of the building foundation are planned in the same way. This proportion is presumed to be fairly similar to the Tang cubit system 唐尺 (*tangch'ŏk*), which is equivalent to 29.694 cm, or the length of the *cha*, a unit of measurement used during the Unified Silla era. Yoneda Miyoji discovered the adoption of the Tang cubit (29.694 cm) by conducting an investigation of Sŏkkuram Buddhist Grotto and Pusŏksa Monastery. The measurement was different from the length of the *cha* used at that time, which was identical to a *kokch'ŏk* 曲尺 (30.303 cm), and implies the use of a carpenter's try square. Measuring each stone element of the Buddhist temples, he noted that the same measurements, equal to 0.98 *kokch'ŏk*, 1.96 *kokch'ŏk*, and 23.6 *kokch'ŏk*, were used repeatedly. He then concluded that the *cha* used by architects and stonemasons at the time of Unified Silla was equal to 0.98 *kokch'ŏk*, i.e. 29.694 cm, and that it was a unit of measurement used in the Tang dynasty. He named the *cha* the Tang cubit, the reference scale of the Tang Dynasty (Yoneda 1944, 1976, 26–28). Units of length in Chinese measurements were rooted in human dimensions. These origins were comparable to those of Greek metrology; however, the Greek and Romans preferred the foot as a unit of measurement, while the Chinese, Korean, Egyptians, Ancient Indians, and Mesopotamians preferred the cubit based on the forearm length from the tip of the middle finger to the bottom of the elbow. The other Unified Silla constructions built at the sites of the Mangdŏksa Monastery, Sach'ŏnwangsa Monastery, and anonymous Buddhist site in Ch'ŏn'gulli District employed the Tang cubit (Kim 2007, 456). Likewise, in the Isŏng sansŏng Mountain Fortress, Hanam City, Kyŏnggi Province, a wooden ruler estimated to have been used in the Silla era before and after the unification of the Three Kingdoms in the 7th century was excavated in 1999 (*Chungang ilbo* 1999, December 21). The ruler, 29.8 cm in total length, had nine graduations engraved on the side with equal spacing, which followed an identical measurement system to the Tang standard ruler, and has been universally used in East Asian countries since its adoption during the Tang dynasty. It is interesting to note that the numerical value of 21 times is produced without error when 29.694 cm based on the Tang cubit is divided by √2 (= 1.414).[14] Thus, the Tang cubit and √2 make a very good pair and merit future study, given that a Tang cubit ruler in the Kyŏngju capital during the Unified Silla period has yet to be discovered.

A cross-section of the Muryangsujŏn Hall shows the numerical value of √2 plus the concept of an arithmetical and geometric sequence more clearly. It can be seen that the basic ground for designing the building is the height of the front and back eaves columns with column-top bracketing, and the height of the eaves purlins as well. On the premise that the height of the eaves columns is one (1), the height of the eaves purlins is √2 times. To put it plainly, the authors can draw a square corresponding to the height of the eaves columns, as well as an inscribed circle and circumscribed circle along the corresponding vertices of the square. The eaves purlins then meet at the point where the radius of the circumscribed circle becomes √2 times. From a practical view through measured values, the height of the eaves columns is 3,460 mm, and the vertical length from the upper side of the building platform to the lower side of the eaves purlins is 4,846 mm. √2 times the height of the eaves columns in cross-section is 4,892 mm. This represents an error of approximately 1% at the distance of 4,846mm to the lower side of the eaves purlins. It is judged that the other figures cannot be confirmed in the current line drawings. However, the most remarkable thing is that both the √2 times the length between the central bay and the outermost bays along the front façade of the ground plan and the ratio of actual height of the eaves columns to the height of the eaves purlins share a margin of error less than two percent in comparison with the actual measurements (Cultural Heritage Administration of Korea 2002, 133).

Interesting results are in the fact that the Muryangsujŏn Hall is a building made of wooden components with shrinkage and expansion due to seasonal temperature change in process of time, together with the maintenance records of dismantlement and repair works. Besides, the criteria are the vertical distance from the upper side of the building platform to the lower side of the eaves purlins, located between intermediary bracket-like timbers (between the lead bracket arm (*linggong*) and the purlin) and eaves rafters. In tune with that of the Muryangsujŏn Hall, as seen through Wang's research (Wang 2011a, 2011b), Chinese wooden architecture applies √2 times the vertical distance from the upper side of the building platform to the upper side of the eaves purlins. This shows that there is a definitive difference between the proportional standards of Korean and Chinese architecture, although the rule of √2 times is equivalently applied at a regular distance. These distinctions appear differently, drawing on country, region, and period. (Figure 7)

Additionally, the height of the eaves columns is a standard distance when compared to various internal elements. The height of the interior columns is √2 times

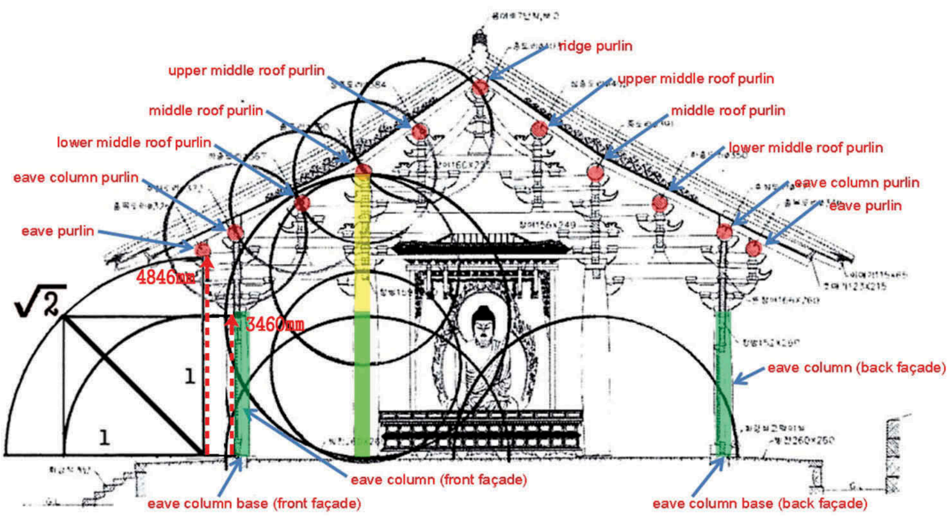

Figure 7. Cross-section of the Muryangsujŏn Hall (Cultural Heritage Administration of Korea 2002, 154).

[14]The Tang cubit ruler, 29.694cm, which was used during the Unified Silla period, is a generalized theory widely used among Korean scholars. In addition, according to Zhang Shiqing, a Chinese scholar, he confirms that there are twenty-six examples of Tang cubit ruler and Tang cubit ruler at Shoshoin 正倉院 in Japan, most of which are measured between 29.5 ~ 29.7cm in length. (Zhang 2004, 77) Therefore, it can be seen that the Tang cubit ruler used in the Korean Peninsula was ratified to 29.694cm, which is intimately associated with 21 times the numerical value of √2. In order to verify the correlation with the ratio of 21 times and the square root of 2, more detailed researches are needed in the future.

when compared with that of the eaves columns. Double the height of the eaves columns is equal to the distance from the interior column bases to the middle roof purlin. The elevation of the middle roof purlin measures twice the height of the lower eaves column. The proportional interdependence between the middle roof purlin and the eaves columns is currently revealed not only in the two Tang buildings (the Main Hall of Nanchansi Monastery and the East Hall of Foguangsi Monastery at Mount Wutai),[15] but also in Liao and Song buildings, including the late 10th century Guanyinge Pavilion of Dulesi Monastery, the early 11th century main hall of Baoguosi Monastery (Ningbo, Zhejiang), the mid-11th century main hall of Shanhuasi Monastery (Datong, Shanxi), and so on (Fu, Steinhardt, and Harrer 2017, 209–214; Fu 1998, 147–167). The application of this principle appears in Unified Silla construction. (Figure 8) The Sŏkkuram Grotto's vertical scheme shows that the carvings of the eight Bodhisattvas on the shrine enclosure, including their pedestals, stand 12 *cha* high. The height from the carvings to the shrine enclosure's ceiling is 12 times the square root of 2. (Yoneda 1944, 1976, 27–28, 140–141) (Figure 9)

The height of the eaves columns is also relevant to the depth between each purlin. The depth of a building is usually measured by the number of rafter lengths necessary to cover the building from front eaves columns to rear eaves columns, and is equivalent to the spaces between the purlins in cross-section. The distances from the center of the middle roof purlin to the center of the ridge purlin, and from the center of the middle roof purlin to the center of the eaves purlin are the same as the height of the eaves column. Although the distance between each purlin is uniformly distributed, the distance between each purlin center could be different due to the different tilt of each purlin; however, the distance between the centres of each purlin is usually the same, excluding the eaves purlin. A feature seen more clearly on the cross-section than on the ground plan is the fact that the distance from the front interior column to the back (the distance from the front high column to the back inside the hall) is equivalent to the distance from the interior high column to the side border of the building foundation. The height of the eaves columns is closely related to the length of each interior element, and the intimate relevance can be found in several parts. When multiplying the height of the eaves columns by $\sqrt{2}$, it matches the position of the eaves purlin. The height of the eaves columns is the same distance from the center of the columns purlin to the center of the middle roof purlin. It is also equal to the distance from the outline surface of the interior columns to the eaves columns. In addition, half of the height from the interior column bases to the middle roof purlin meets the outline surface on the exterior columns. What is remarkable is not the distance to the eaves column center, but the distance to the outline surface of the eaves column. To put it another way, if $\sqrt{2}$ times the height of the eaves column is multiplied $\sqrt{2}$ times, then the height of the eaves columns is doubled, and corresponds to the location of the middle roof purlin. This is a demonstration of the concept of geometric progression. Multiplying this doubled height by $\frac{1}{2}$ is equal to the distance from the eaves columns to the interior columns. The height of the eaves pillars is equal to the distance from the center of the ridge purlin to the center of the middle roof purlin, and the distance from the center of the middle roof purlin to the columns purlin.

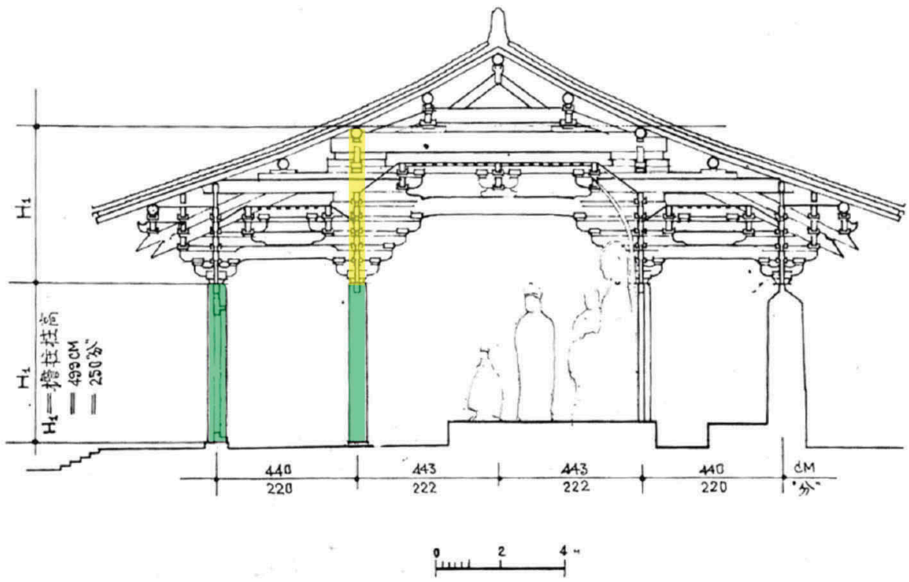

Figure 8. Cross-section of the Dongdadian (the Great East Hall) at Foguangsi Monastery (Fu 1998, 153).

[15]In the case of the main hall of Nanchansi Monastery, the eaves columns are about 3.86m high, while the vertical distance between the eaves column top and the ridge purlin is about 3.81m high. Since this hall is only four rafters deep, the ridge purlin corresponds to the middle roof purlin in cross-section. A similar example can be seen in the main gate of Dulesi Monastery.

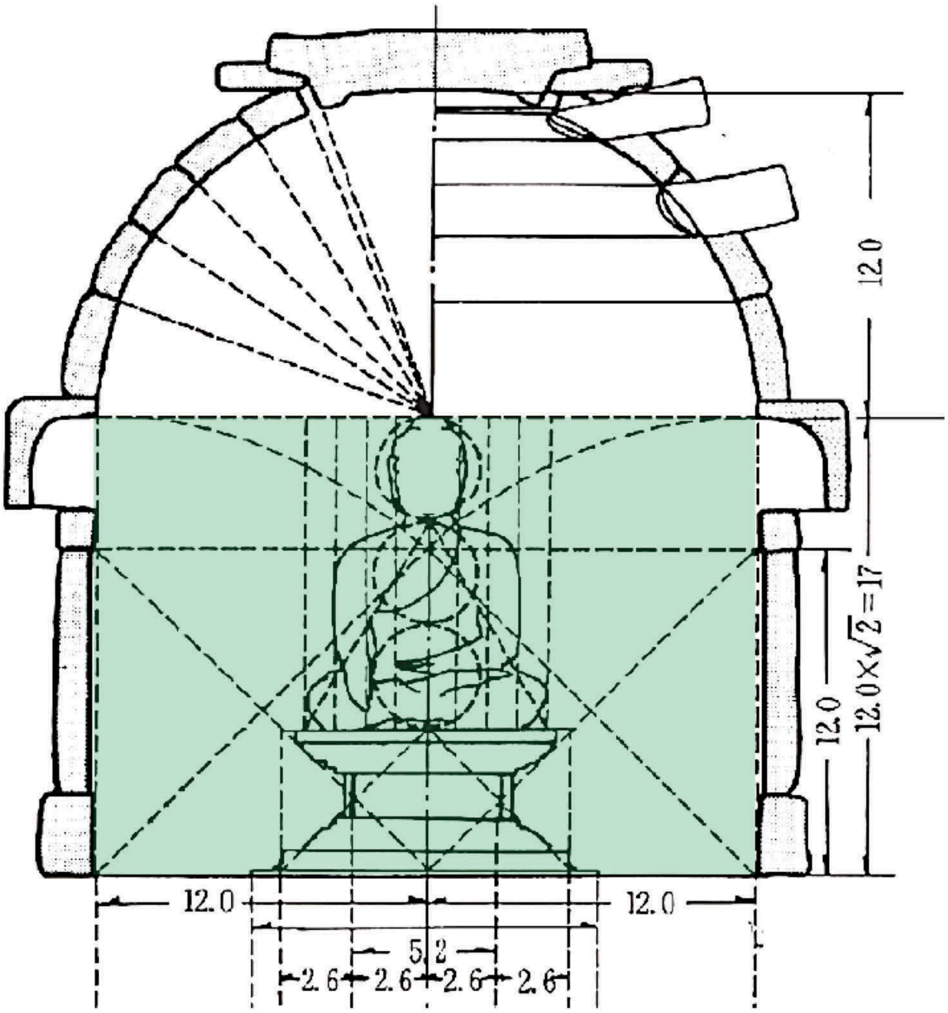

Figure 9. Systematic proportion system in the vertical scheme of Sŏkkuram Grotto (Yoneda 1944, 1976, 140–141).

It can also be applied to the distance from the columns purlin to the lower edge of the roof eaves. From the cross-sectional profile, the distance from the eaves column bases to the interior column tops is similar to the distance from the eaves column bases to the ends of the flying rafters. Moreover, when the height of the eaves columns is multiplied √2 times, it is equal to the distance from the eaves column tops to the interior column bases. Applying √2 times or twice the height of the eaves columns reveals an intimate connection with the distance from the one column to the other, from the column tops to the building foundation, and from the column bases to the ends of the eaves purlins. This is to say that the measurement system is based on the Tang cubit, which was widely used during the Unified Silla period. In both the ground plan and the cross-section, √2 times and twice the height or distance of the eaves columns, depending on the distance between each column and the distance between each purlin, is applied as a regular proportional concept in constructing a building.

Thus, the proportional concept of √2 times and two times can be seen as an important proportional concept in the construction of interior frame structures for wooden architecture. Through the proof of √2 times the height ratio obtained from the Muryangsujŏn Hall at Pusŏksa Monastery, there should be numerous references to aid in their reconstruction on the ruined monastic sites of the Kyŏngju Historic Areas, which are supposed to be constructed with the Tang cubit of the Unified Silla period. However, since the height of the column cannot be confirmed at the ruined monastic sites, the method for estimating the height of the columns by judging the width of the stone plinths and the distance between each plinth on the center should be studied in the future. The fact that the concept of √2 numerical value is found in connection with wooden elements from the Muryangsujŏn Hall, which is recognized as a Koryŏ building in Korean Peninsula, should be further complemented by an effort to compare it with such coincident and related fields of study as wood-frame construction in ancient East Asian civilization.

4. Discussion

By dwelling upon the philosophical background of ancient mathematics references in ancient China, the *Zhoubi Suanjing* and *Jiuzhang Suanshu*, this paper analyses the proportional system of interior timber-frame structures, with the following results. First, from among the contents of the mathematics books in ancient China, this paper shows that it is possible to represent all things as circles and rectangles, and to replace circles and rectangles with each other, in connection with the ancient ideation, *Tianyuan difang* (Heaven is round and Earth is square), a philosophical background in architecture and urban planning that was very deeply reflected in formative thought. Second, the proportional system of inscribed and circumscribed circles made by circles and squares on a ground plan has a very close link with the ratio of 1 and √2 (1.414) in shaping a wooden framed structure. These numerical values can be important criteria for the composition of the square ground plan and the cross-section. Third, the ratio of the height of the eaves columns to the height of the eaves purlins at the Muryangsujŏn Hall is 1: √2, and similarly, that of the height of the eaves columns to the interior columns is 1: √2. In some cases, it can be 1: 2, dependent on the height of the ridge purlin and the width of the span of the roof. The distance ratio between each purlin and column produces arithmetic and geometric proportions. Especially, the height of the eaves columns is found to be a reference value for the whole framed structure. Fourth, this thesis reconfirms that Pusŏksa Monastery is built with the Tang cubit (29.694cm) in a strong proportional consideration with the square root of 2. In dividing 29.694 cm by 1.414, the natural number of 21 times appears as a numerical value; the Tang cubit thus seems to have a deep connection to 1.414. Fifth, the Monastery Pusŏksa's Muryangsujŏn Hall has been recognized as a Koryŏ monument established between the 11th and 13th centuries through the combination of the findings in this article and the previous studies that review structural attributes: architraves without any extended protuberances through the pillars at the corners; diagonal beams amalgamated with corner bracket complexes; repetitive application of wooden components with regular size; acceptance of the pre-Koryŏ's methods for a stone platform; and comparison and analysis on the architectural styles with the Chosadang Hall built 1377 as well (Sekino 1941; Chŏng 1974; Munhwajaegwalliguk 1980; Han'gukpulgyoyŏn'guwŏn 1988; Han 2002; Cha 2014; Kim 2011, 2014).

All these things make it clear that the Muryangsujŏn Hall conforms to the old building method because its quintessential design notion can be explained with the contents of the round-square and square-circle maps that apply √2 times as the key ratio in the *Zhoubi Suanjing* and *Jiuzhang Suanshu*. The Koryŏ builders followed East Asian dictates from base to roof in order to create symbolic buildings with an association of power. The foundation platform and stone plinths of the Muryangsujŏn Hall are believed to date from the late Unified Silla period. The diagonal beams combined with the bracket sets above the columns at the four corners of the Muryangsujŏn Hall can be meaningful shreds of evidence that the Buddhist monument performed a repair work in the 14th century preserving the 11th century's original appearance reconstructed after the fire, so it is expected that its specific construction era will be further reconsidered through future research efforts by comparison with other extant timber-frame buildings across East Asia.

Disclosure statement

No potential conflict of interest was reported by the authors.

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